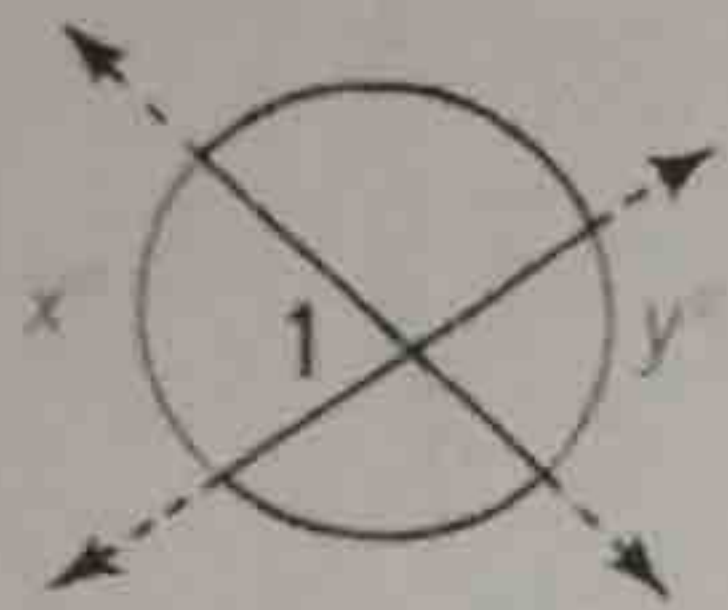


8.4 Angle Measures and Segment Lengths

SWBAT apply the rules and theorems of segments to solve for unknowns.

Theorem 1:

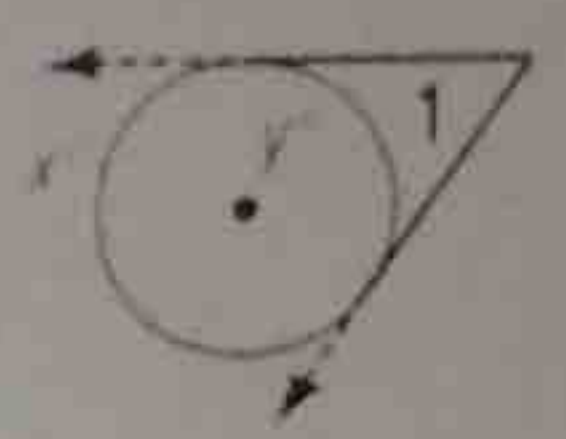
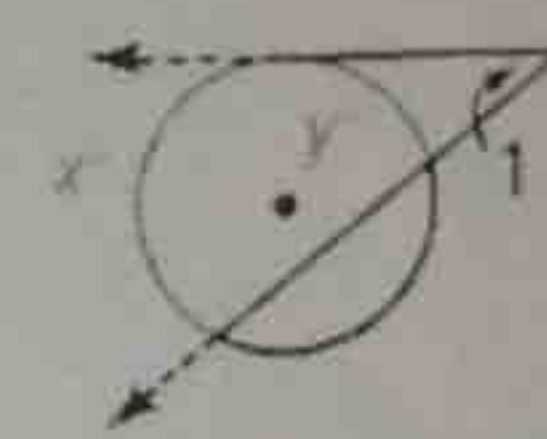
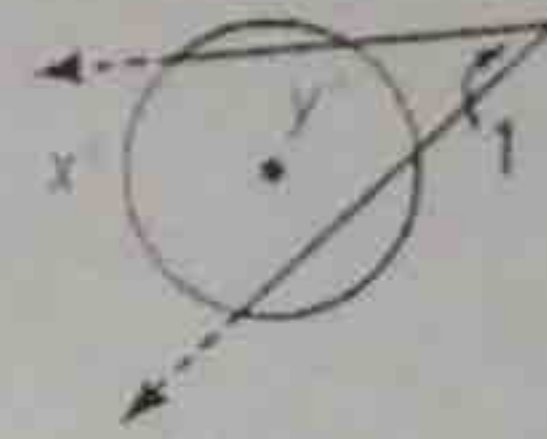
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.



$$m\angle 1 = \frac{1}{2}(x + y)$$

Theorem 2:

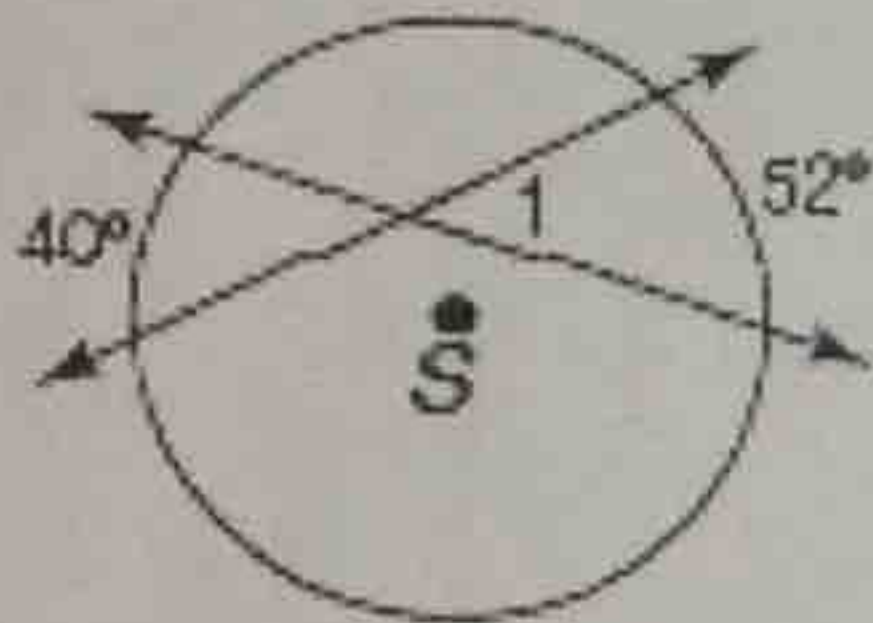
The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.



$$m\angle 1 = \frac{1}{2}(x - y)$$

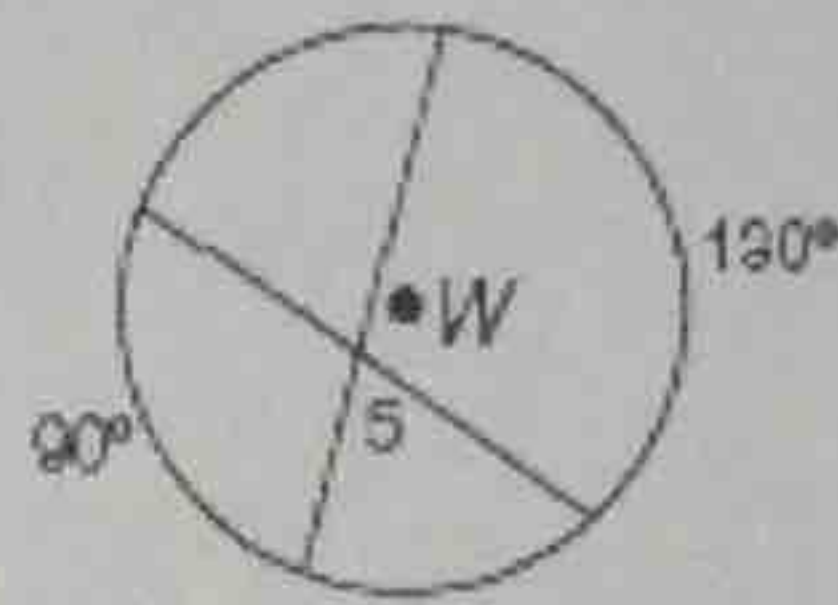
Example 1: Find each measure.

a) $m\angle 1$



$$m\angle 1 = \frac{52 + 40}{2} = 46^\circ$$

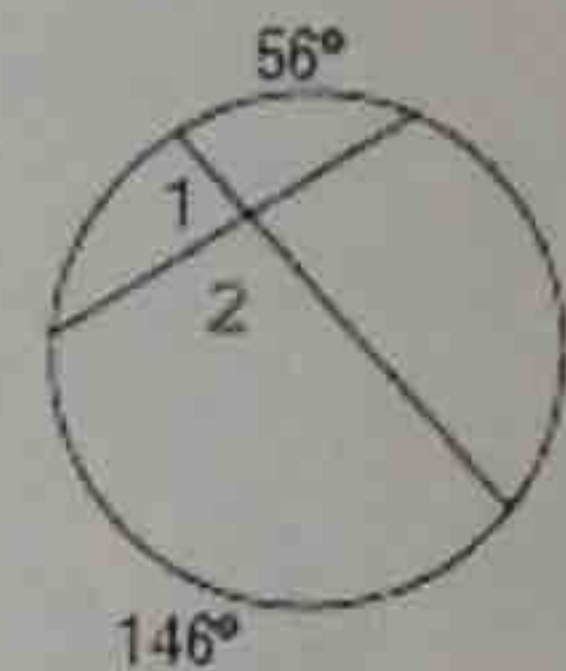
b) $m\angle 5$



$$m\angle 5 = \frac{90 + 130}{2} = 110^\circ$$

$$m\angle 5 = 180 - 110 = 70^\circ$$

c) $m\angle 1$

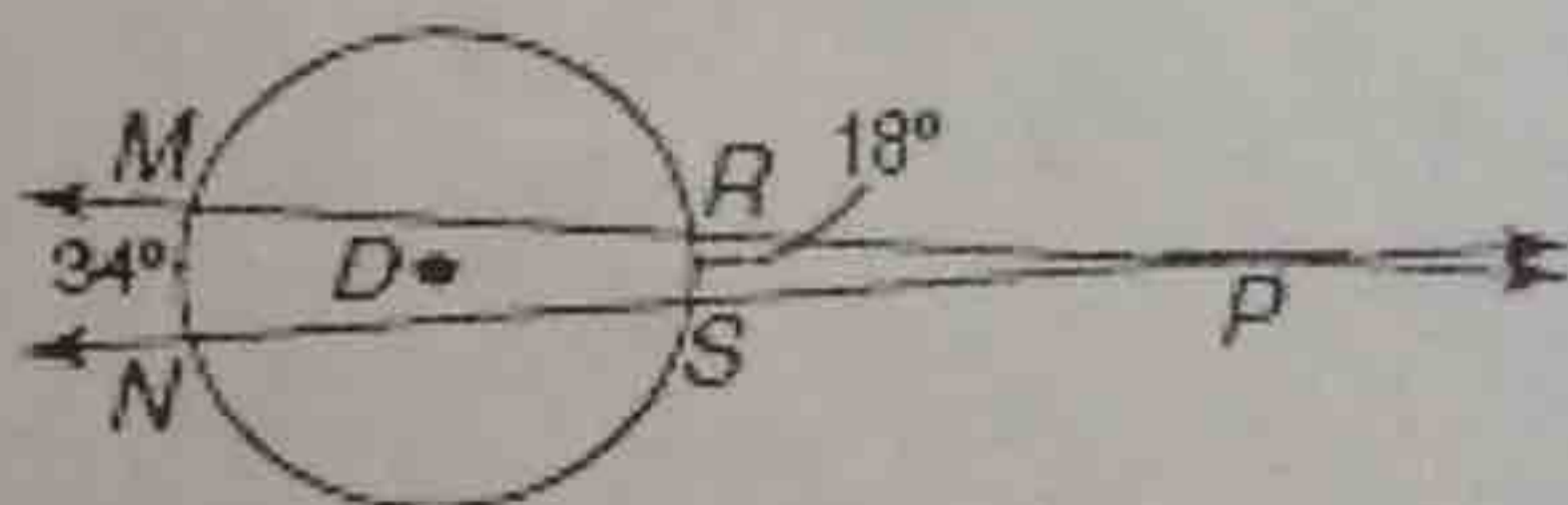


$$m\angle 2 = \frac{146 + 56}{2} = 101$$

$$m\angle 1 = 180 - 101 = 79^\circ$$

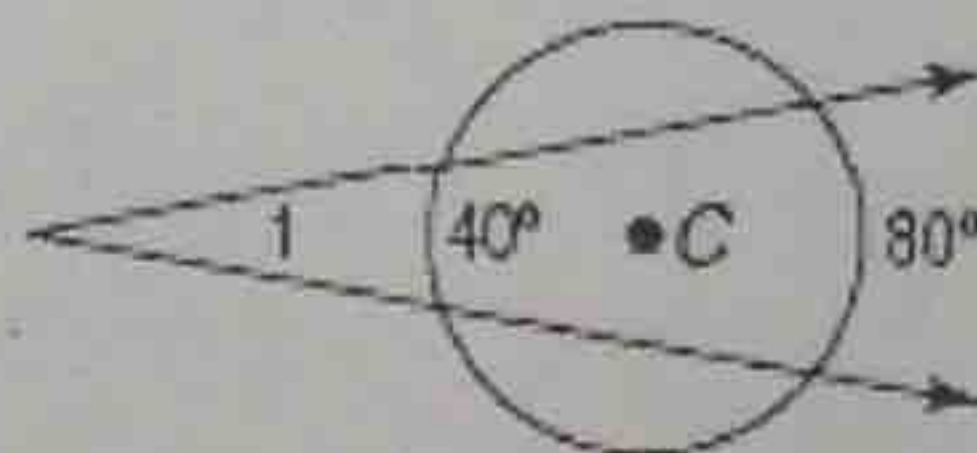
Example 2: Find the following angles.

a) $m\angle MPN$



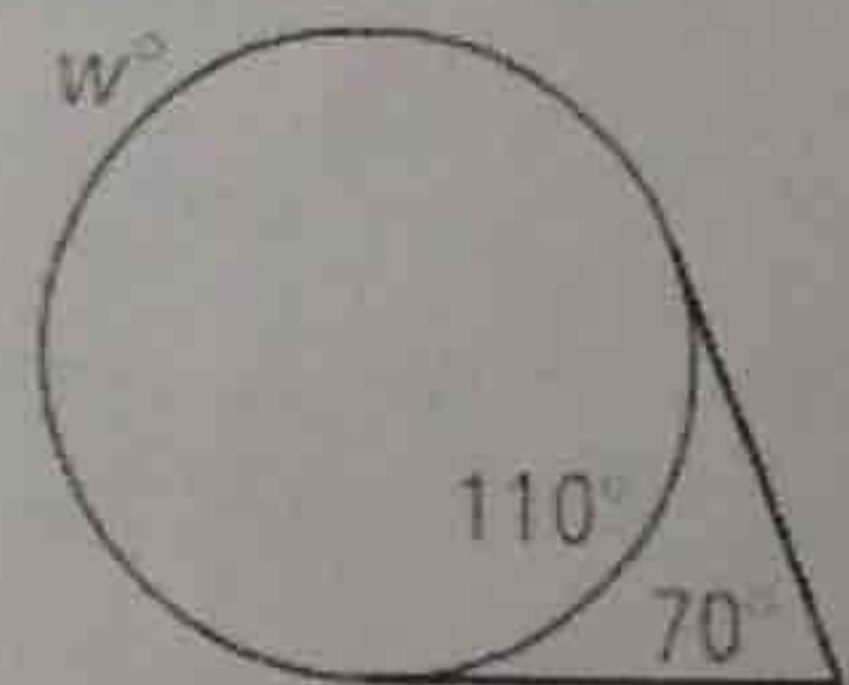
$$m\angle MPN = \frac{34 - 18}{2} = 8$$

b) $m\angle 1$



$$m\angle 1 = \frac{80 - 40}{2} = 20^\circ$$

c) w°



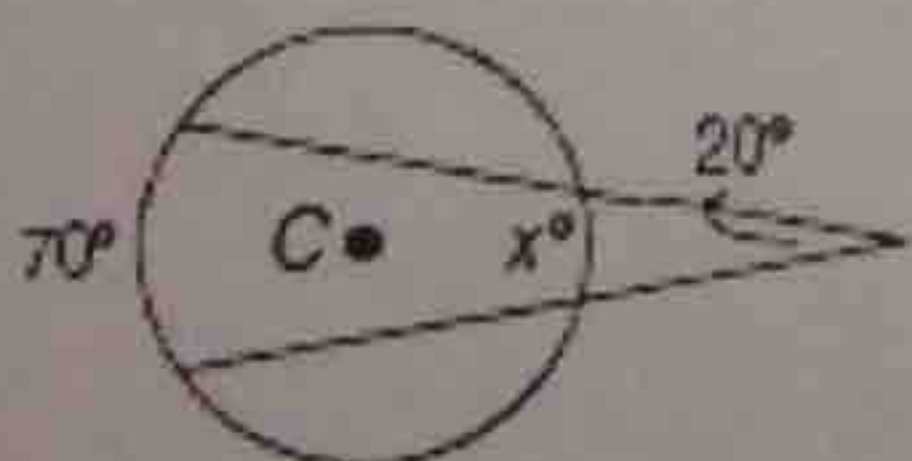
$$70 = \frac{w - 110}{2}$$

$$140 = w - 110$$

$$w = 250^\circ$$

You Try! Find the following angles.

a) x

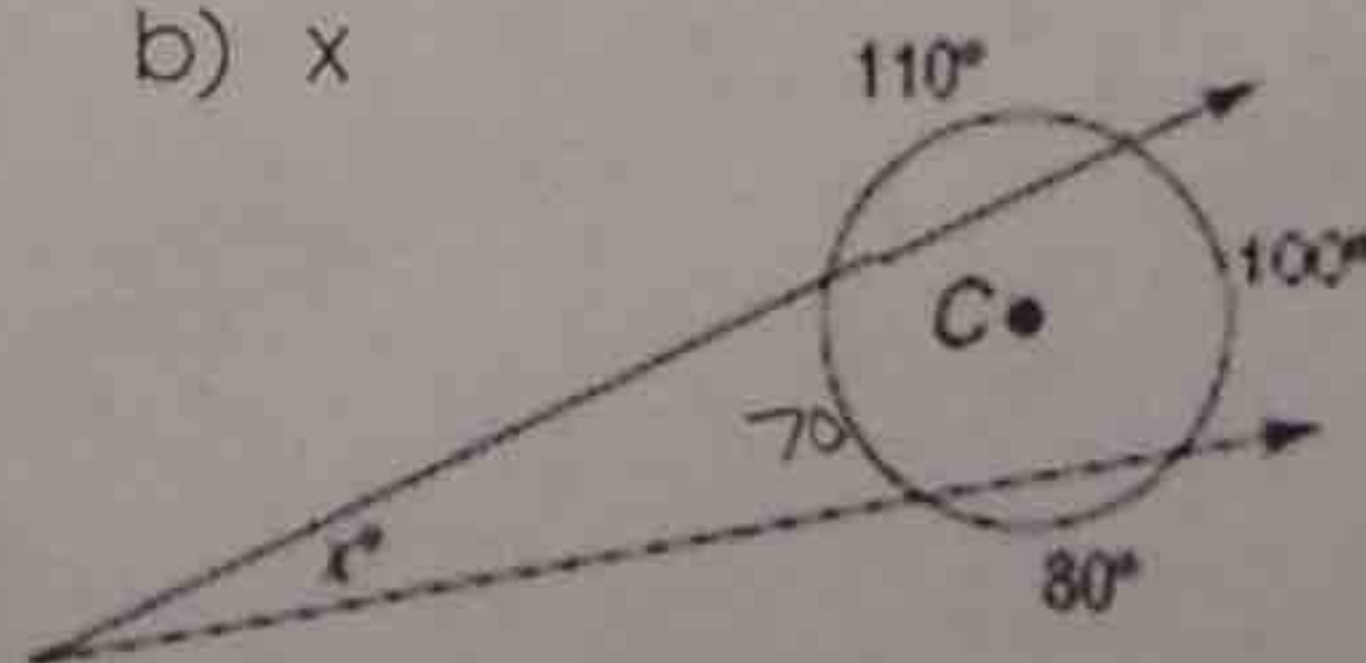


$$\frac{70 - x}{2} = 20$$

$$70 - x = 40$$

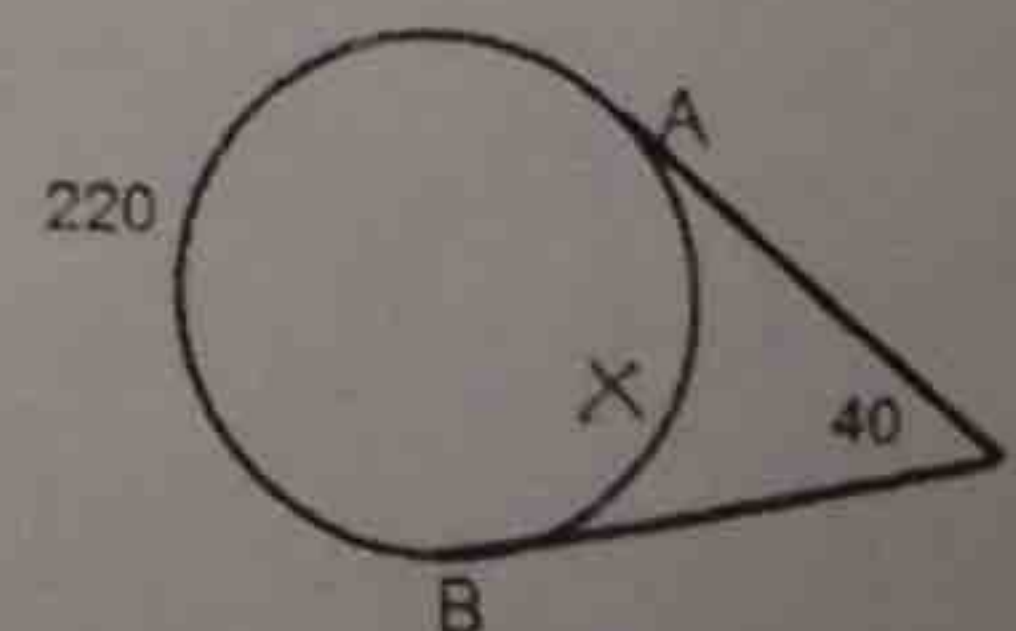
$$x = 30$$

b) x



$$x = \frac{100 - 70}{2} = 15^\circ$$

c) Arc AB



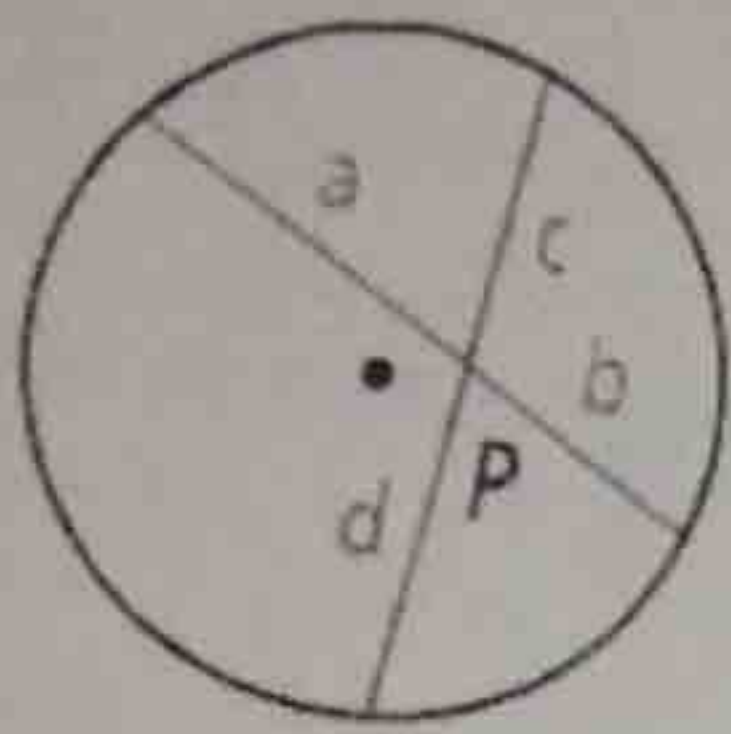
$$40 = \frac{220 - x}{2}$$

$$80 = 220 - x$$

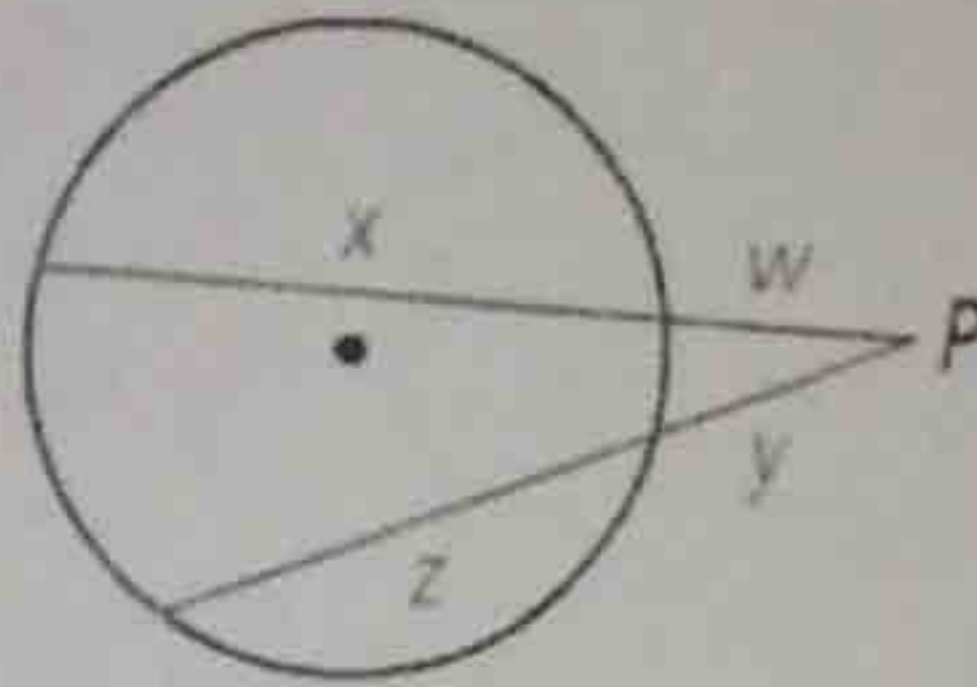
$$x = 140^\circ$$

Theorem 3:

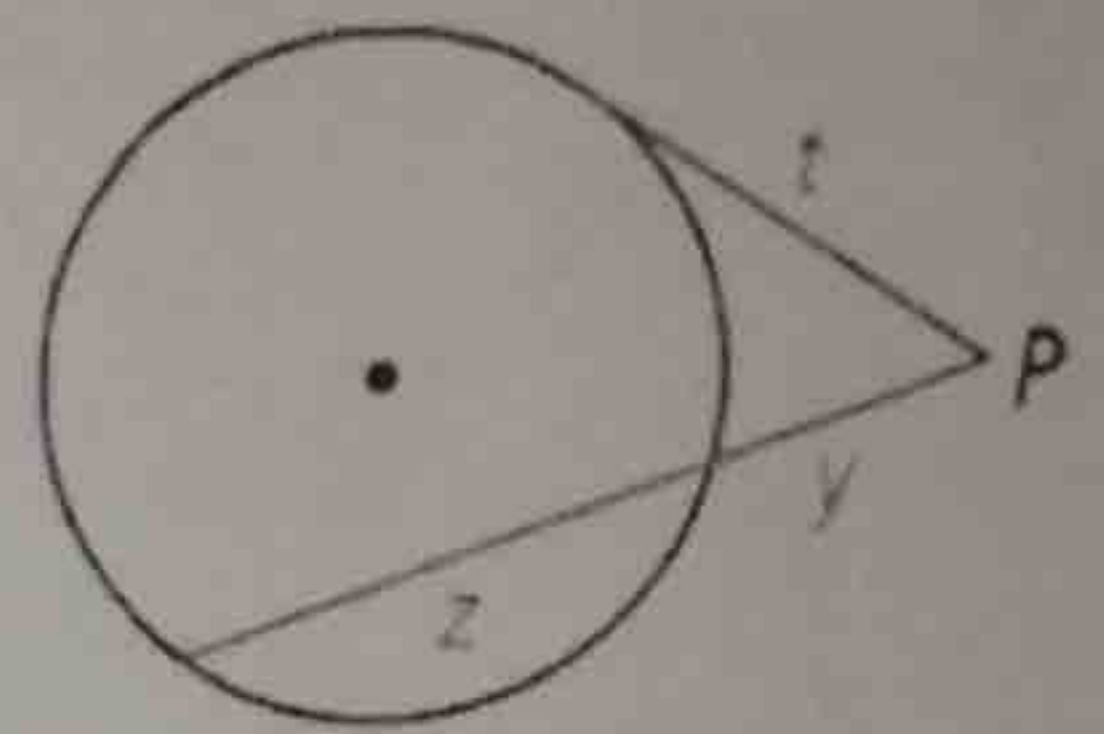
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and the circle.



$$a \cdot b = c \cdot d$$

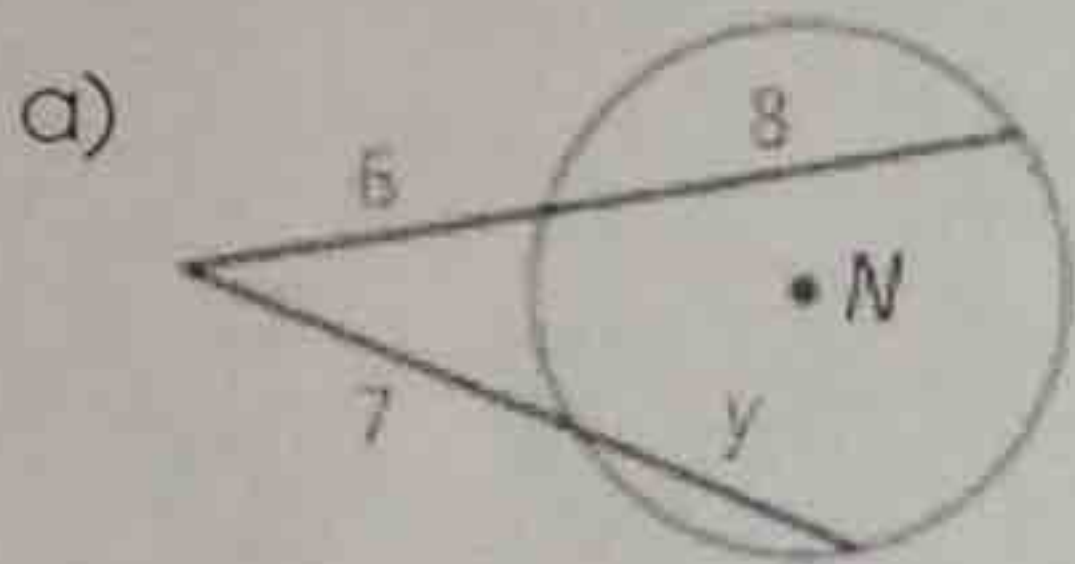


$$(w + x)w = (y + z)y$$

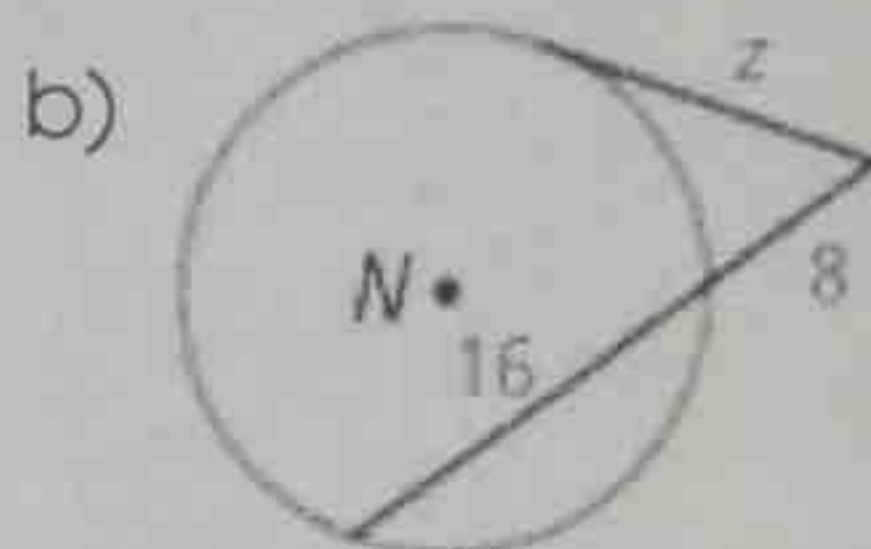


$$(y + z)y = t^2$$

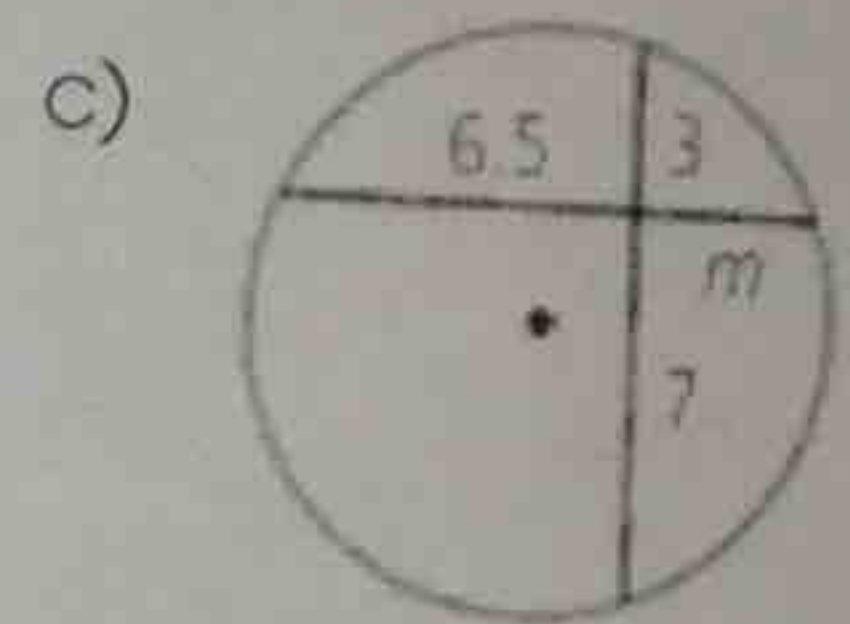
Example 4: Find the value of the variable in $\odot O$.



$$\begin{aligned} 6(6+8) &= 7(y+7) \\ 84 &= 7y + 49 \\ 35 &= 7y \quad y = 5 \end{aligned}$$



$$\begin{aligned} z^2 &= 8(16+8) \\ z^2 &= 192 \\ z &= 13.9 \end{aligned}$$



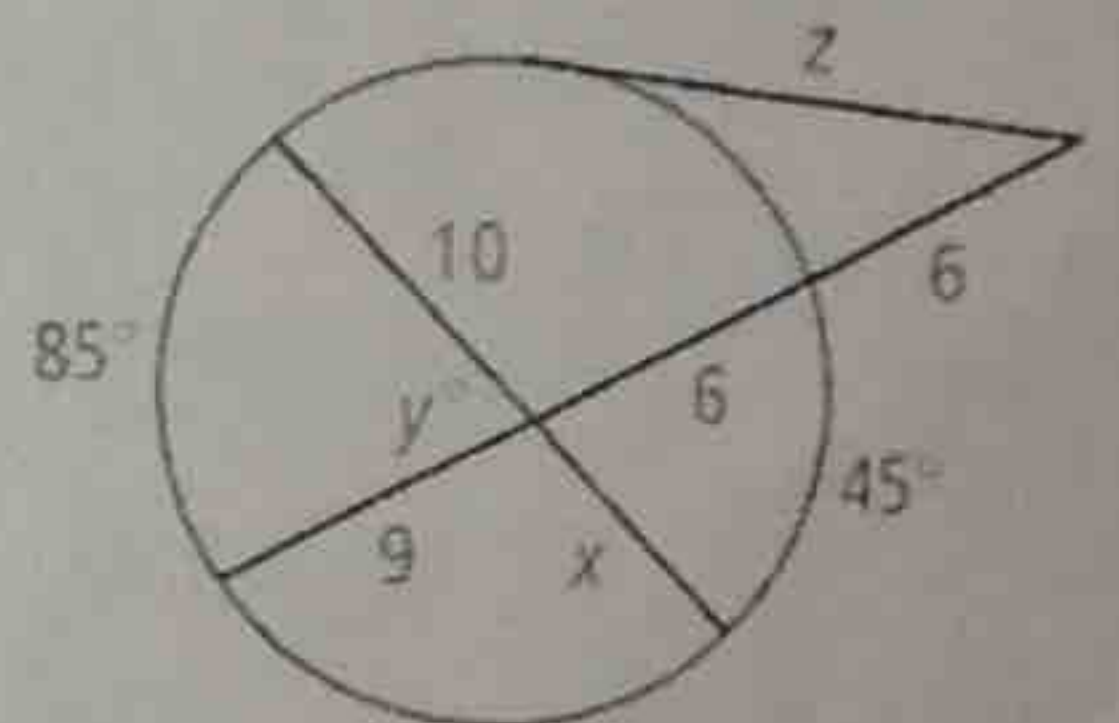
$$\begin{aligned} 3(7) &= m(6.5) \\ 21 &= 6.5m \\ m &= 3.2 \end{aligned}$$

You Try! What is the value of the variable to the nearest tenth?

$$\begin{aligned} 9(6) &= 10x \\ 54 &= 10x \\ x &= 5.4 \end{aligned}$$

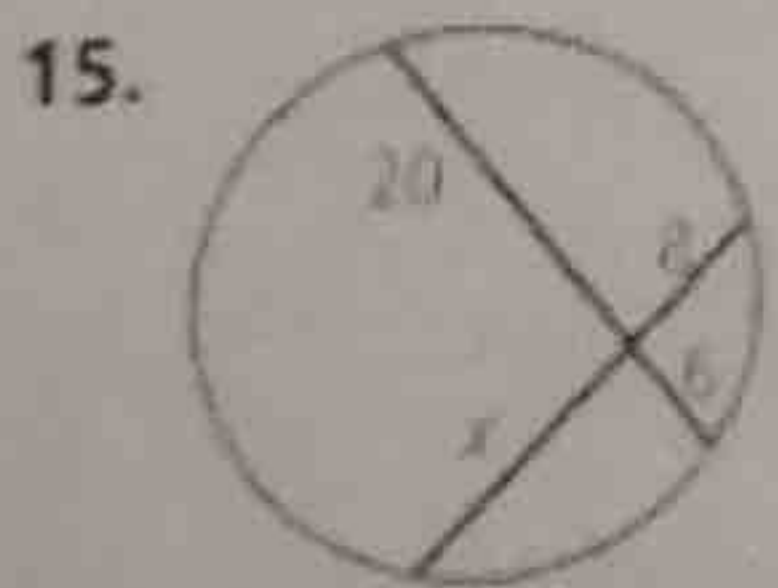
$$\begin{aligned} y &= \frac{85+45}{2} \\ y &= 65 \end{aligned}$$

$$\begin{aligned} z^2 &= 6(6+15) \\ z^2 &= 126 \\ z &= 11.2 \end{aligned}$$

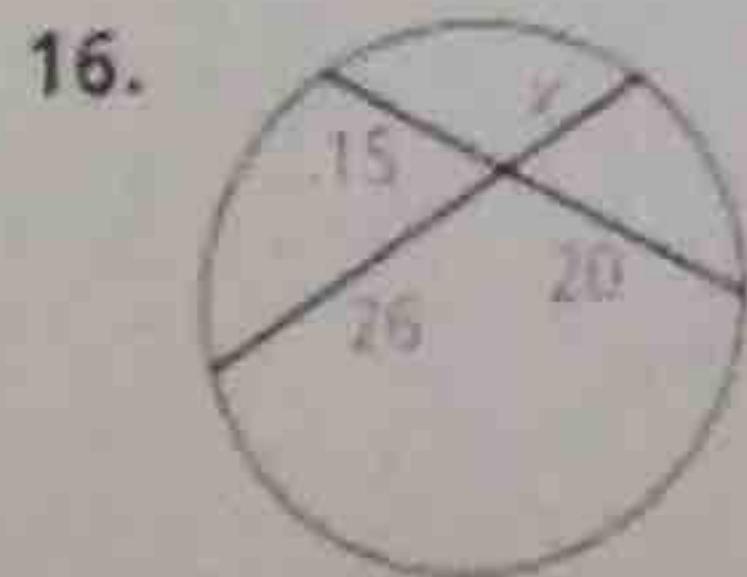


Algebra Find the value of each variable using the given chord, secant, and tangent lengths. If the answer is not a whole number, round to the nearest tenth.

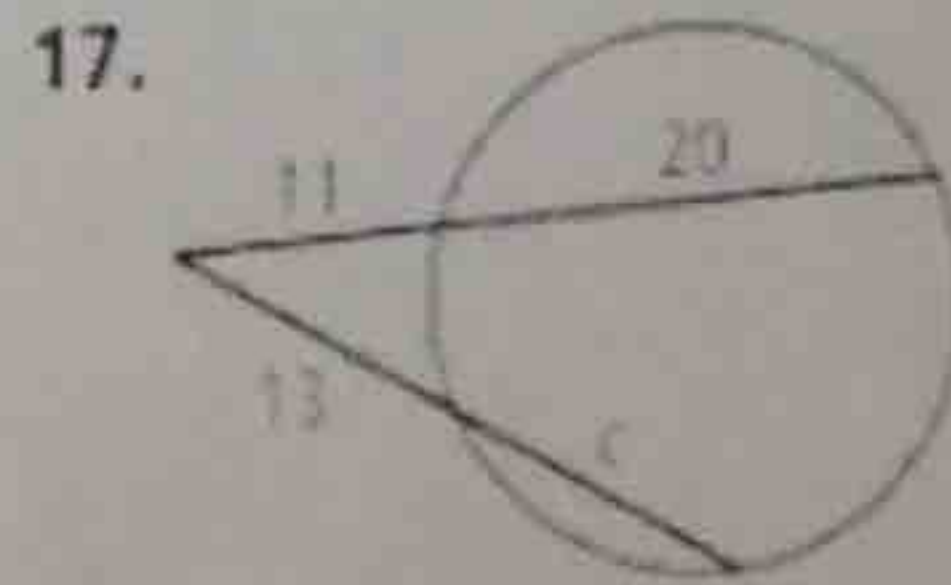
See Problem 3.



$$\begin{aligned} 8x &= 120 \\ x &= 15 \end{aligned}$$

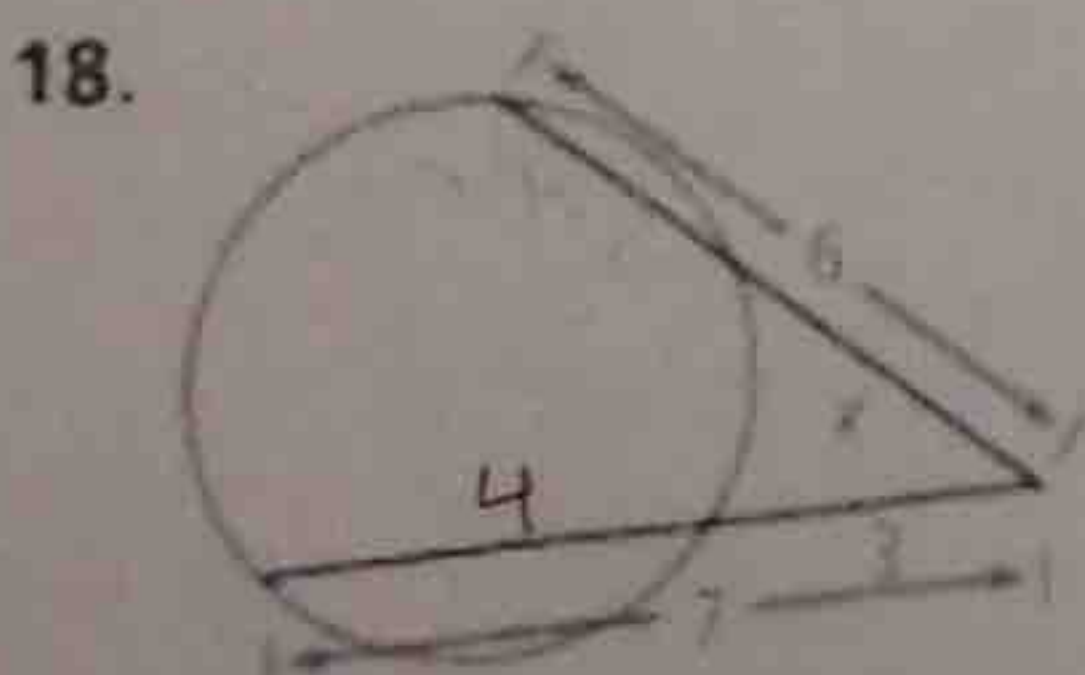


$$\begin{aligned} 26x &= 300 \\ x &= 11.5 \end{aligned}$$

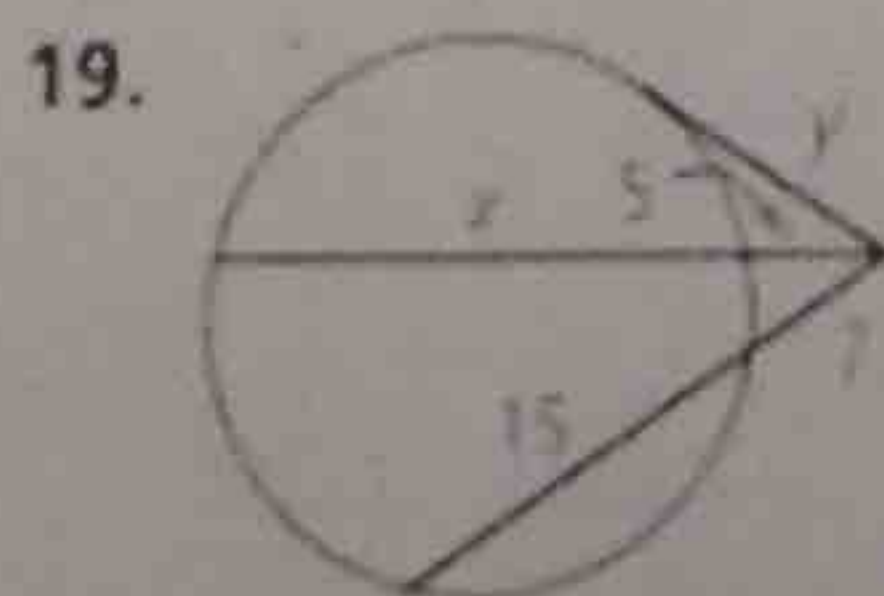


$$\begin{aligned} 13(c+13) &= 11(11+20) \\ 13c+169 &= 341 \end{aligned}$$

$$\begin{aligned} 13c &= 172 \\ c &= 13.2 \end{aligned}$$

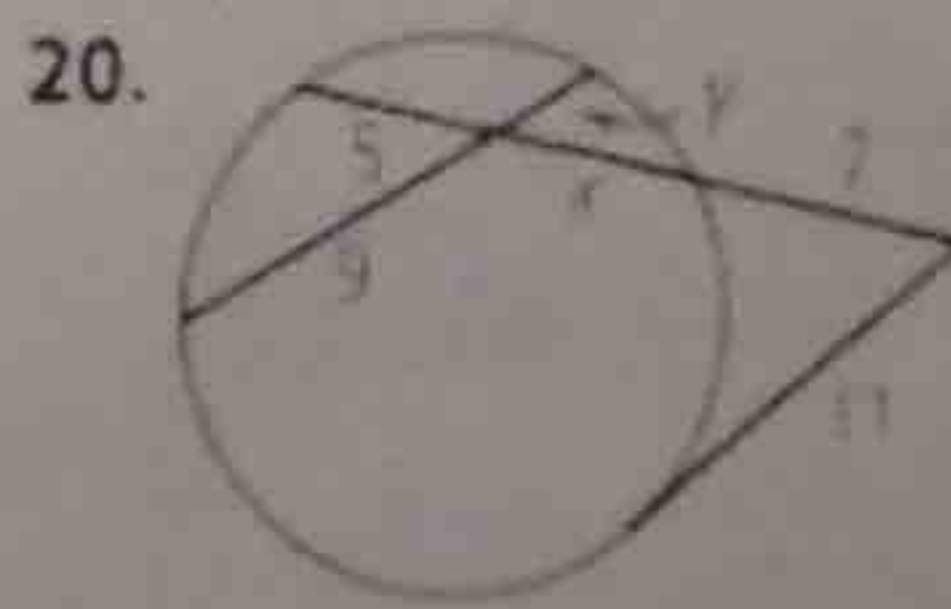


$$\begin{aligned} 3(7) &= x(6) \\ 21 &= 6x \\ x &= 3.5 \end{aligned}$$



$$\begin{aligned} 7(22) &= 5(x+5) \\ 154 &= 5x+25 \\ 5x &= 129 \\ x &= 25.8 \end{aligned}$$

$$\begin{aligned} 154 &= y^2 \\ y &= 12.4 \end{aligned}$$



$$\begin{aligned} 11^2 &= 7(x+12) \\ 121 &= 7x+84 \\ 7x &= 37 \\ x &= 5.3 \end{aligned}$$

$$\begin{aligned} 5(5.3) &= 9y \\ 26.5 &= 9y \\ y &= 2.9 \end{aligned}$$